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The Quantum Fluctuations of Mesoscopic Damped Mutual Capacitance Coupled Double Resonance RLC Circuit in Excitation State of the Squeezed Vacuum State

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Mesoscopic damped double resonance mutual capacitance coupled RLC circuit is quantized by the method of damped harmonic oscillator quantization. The Hamiltonian is diagonalized by unitary transformation. The eigenenergy spectra of this circuit are given. The quantum fluctuations of the charges and current of each loop are researched in excitation state of the squeezed vacuum state, the squeezed vacuum state and in vacuum state. It is show that, the quantum fluctuations of the charges and current are related to not only circuit inherent parameter and coupled magnitude, but also quantum number of excitation, squeezed coefficients, squeezed angle and damped resistance. And, because of damped resistance, the quantum fluctuation decay along with time.

KEY WORDS: mesoscopic circuit; damped double resonance RLC circuit; unitary transformation; quantum fluctuation.

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1. INTRODUCTION

With the development of manometer techniques and microelectronics, the trend of the miniaturization of integrated circuits and electronic components becomes stronger and stronger. When the phase coherence length of the charge-carrier approaches the Fermi wavelength, the quantum effects must be taken into account. Louisell (1973) first discussed the quantum effects of an LC circuit and gave its quantum noise in the vacuum state. Afterward, the quantum effects of circuit and device have become one of the hotspot in mesoscopic physics. A lot

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of literatures (Wang and Sun, 1998; Yu and Liu, 1998; Zhang, 1998; Fan and Liang, 2000; Wang *et al.*, 2000; Ji and Lei, 2001; Ji, 2002; Wang, 2002; Liu *et al.*, 2003; Song, 2003; Li, 2005) indicated that quantum fluctuations of the electric charges and current was studied widely in mesoscopic circuit. A lot of researches on quantum effects in nondissipative mesoscopic circuits had been done. Recently, Some researches in influence of a resistance on quantum effects in the dissipative mesoscopic circuit have been done. We know that the harmonic oscillator model is more significant in quantum mechanics. If a actual physic problem can be equivalent with quantized harmonic oscillator, it is convenient to solve this problem. But using the method of damped harmonic oscillator quantization and unitary transformation, the article of the quantum fluctuations of mesoscopic damped double resonance mutual capacitance coupled circuit has not been published yet. As the squeezed vacuum state and the vacuum state can be regarded as special cases of the excitation state of the squeezed vacuum state, it is more general and significant to study quantum in the charge and current in excitation state of the squeezed vacuum state. In this paper, mesoscopic damped double resonance mutual capacitance coupled circuit is quantized by the method of damped harmonic oscillator quantization (Peng, 1980). Hamiltonian is diagonalized by the method of unitary transformation. The energy spectra of this circuit are given. Furthermore, the quantum fluctuations of the charge and current of each loop in the excitation state of the squeezed vacuum state, squeezed vacuum state and vacuum state are researched.

2. THE QUANTIZATION OF THE MESOSCOPIC DAMPED DOUBLE RESONANCE MUTUAL CAPACITANCE COUPLED RLC CIRCUIT

The circuit of the Fig. 1 is damped capacitance coupled double resonance RLC circuit. According to Kirchoff's law, the classical equations of motion of the system are

$$L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{1}{C}(q_1 + q_2) + \frac{1}{C_1} q_1 = i_{s1}(t) R_1, \tag{1a}$$

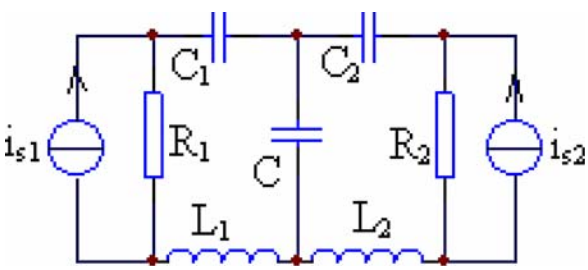

Fig. 1. The damped double resonance mutual capacitance coupled RLC circuit.

$$L_2\frac{d^2q_2}{dt^2} + R_2\frac{dq_2}{dt} + \frac{1}{C}(q_1+q_2) + \frac{1}{C_2}q_2 = i_{s2}(t)R_2, \tag{1b}$$

where $q_u\,(u=1,2)$ are charges of capacitances, $i_{s1}(t)$ and $i_{s2}(t)$ are electric current of message source. To simplify the problem, we suppose that $\frac{R_1}{L_1} = \frac{R_2}{L_2} = \lambda$. Defining $p_\mu = L_\mu\dot{q}_\mu\,(\mu=1,2)$, we have

$$\dot{q}_1 = \frac{p_1}{L_1}, \quad \dot{q}_2 = \frac{p_2}{L_2}, \tag{2a}$$

$$\dot{p}_1 = -\lambda p_1 - \frac{1}{C}(q_1+q_2) - \frac{1}{C_1}q_1 + i_{s1}(t)R_1;$$

$$\dot{p}_2 = -\lambda p_2 - \frac{1}{C}(q_1+q_2) - \frac{1}{C_2}q_2 + i_{s2}(t)R_2. \tag{2b}$$

From Eq. (2), we get

$$\frac{\partial\dot{q}_\mu}{\partial q_\mu} + \frac{\partial\dot{p}_\mu}{\partial p_\mu} = -\lambda,\ \frac{d}{dt}[q_\mu, p_\mu] = -\lambda[q_\mu, p_\mu]\ \mu=1,2. \tag{3}$$

Obviously, when $R_1\neq 0,\ R_2\neq 0$, so as $\lambda\neq 0$, the q_μ and $p_\mu\,(\mu=1,\ 2)$ are not conjugate variable in classical conditions in the damped double resonance mutual capacitance coupled RLC circuit. Movement Equation is fixedness by which Eq. (2) is quantized in Heisenberg picture, q_μ and $p_\mu\,(\mu=1,2)$ stand for coordinate operator and momentum operator, respectively. Eq. (3) indicates that the quantum condition must be modified to satisfy the following damped commutation relations

$$[q_\mu, p_\nu] = j\hbar\tau^{-1}e^{-\lambda t}\delta_{\mu\nu},\ [q_\mu, q_\nu] = [p_\mu, p_\nu] = 0.\,(j^2=-1.\mu,\nu=1,2.) \tag{4}$$

Where, the τ stand for the constant of unit time, its function is to pledge consistent dimension of Eq. (4). From not the common canonical variable q_μ and p_μ to the common canonical variable Q_μ and P_μ, we consider the following commutation

$$Q_\mu = q_\mu\tau^{1/2}\exp(\lambda t/2),$$
$$P_\mu = L_\mu\dot{Q}_\mu = (p_\mu + q_\mu L_\mu\lambda/2)\tau^{1/2}\exp(\lambda t/2),\ (\mu=1,2). \tag{5}$$

From Eq. (4) and (5), we can prove that

$$[Q_\mu,\ P_\nu] = j\hbar\delta_{\mu\nu},\ [Q_\mu, Q_\nu] = [P_\mu,\ P_\nu] = 0.(\mu,\nu=1,2). \tag{6}$$

From Eq. (5) and (2), we obtain

$$\dot{Q}_1 = \frac{P_1}{L_1}, \quad \dot{Q}_2 = \frac{P_2}{L_2}, \tag{7a}$$

$$\dot{P}_1 = -L_1\omega_1^2Q_1 - \frac{1}{C}Q_2 + f_1(t);\ \dot{P}_2 = -L_2\omega_2^2Q_2 - \frac{1}{C}Q_1 + f_2(t). \tag{7b}$$

where

$$\omega_1 = \sqrt{\frac{1}{L_1 C} + \frac{1}{L_1 C_1} - \frac{\lambda^2}{4}},\ \omega_2 = \sqrt{\frac{1}{L_2 C} + \frac{1}{L_2 C_2} - \frac{\lambda^2}{4}}. \tag{8a}$$

$$f_1(t) = i_{s1}(t) R_1 \tau^{1/2} \exp(\lambda t/2),\ \ f_2(t) = i_{s2}(t) R_2 \tau^{1/2} \exp(\lambda t/2). \tag{8b}$$

According to Eq. (7) and canonical Hamiltonian equation $\dot{Q}_u = \frac{\partial H}{\partial P_u}$, $\dot{P}_u = -\frac{\partial H}{\partial Q_u}$ $(\mu = 1, 2)$, we obtain

$$H = \frac{P_1^2}{2L_1} + \frac{P_2^2}{2L_2} + \frac{1}{2} L_1 \omega_1^2 Q_1^2 + \frac{1}{2} L_2 \omega_2^2 Q_2^2 + \rho Q_1 Q_2 - f_1(t) Q_1 - f_2(t) Q_2. \tag{9}$$

Where $\rho = \frac{1}{C}$. Introducing generalized destruction and creation operator

$$a_\mu = \frac{1}{\sqrt{2 L_\mu \omega_\mu \hbar}} (L_\mu \omega_\mu Q_\mu + j P_\mu),$$

$$a_\mu^+ = \frac{1}{\sqrt{2 L_\mu \omega_\mu \hbar}} (L_\mu \omega_\mu Q_\mu - j P_\mu).\ (\mu = 1, 2). \tag{10}$$

From Eq. (6) and (10), we can prove that

$$[a_\mu, a_\nu^+] = \delta_{\mu\nu},\ [a_\mu, a_\nu] = [a_\mu^+, a_\nu^+] = 0.\ (\mu, \nu = 1, 2). \tag{11}$$

The Hamiltonian of this system is

$$\begin{aligned} H = {} & \hbar\omega_1 \left(a_1^+ a_1 + \frac{1}{2}\right) + \hbar\omega_2 \left(a_2^+ a_2 + \frac{1}{2}\right) \\ & + \frac{\hbar}{2} \frac{\rho}{\sqrt{L_1 L_2 \omega_1 \omega_2}} (a_1 a_2 + a_1 a_2^+ + a_1^+ a_2 + a_1^+ a_2^+) \\ & - \sqrt{\frac{\hbar}{2 L_1 \omega_1}} (a_1 + a_1^+) f_1(t) - \sqrt{\frac{\hbar}{2 L_2 \omega_2}} (a_2 + a_2^+) f_2(t). \end{aligned} \tag{12}$$

If electrical source is only instantaneously switched on, say, for an infinitesimal time and then switched off, relevant Hamiltonian can be written as

$$\begin{aligned} H = {} & \hbar\omega_1 \left(a_1^+ a_1 + \frac{1}{2}\right) + \hbar\omega_2 \left(a_2^+ a_2 + \frac{1}{2}\right) \\ & + \frac{\hbar}{2} \frac{\rho}{\sqrt{L_1 L_2 \omega_1 \omega_2}} (a_1 a_2 + a_1 a_2^+ + a_1^+ a_2 + a_1^+ a_2^+). \end{aligned} \tag{13}$$

Eq. (13) is Hamiltonian of typical double coupled harmonic oscillator. When mesoscopic parallel damped double resonance capacitance coupled circuit is quantized, it is equivalent with quantized coupled double harmonic oscillator.

3. HAMILTONIAN WAS DIAGONALIZED

In order to study quantum fluctuations of the mesoscopic damped double resonance mutual capacitance coupled RLC circuit. Above all, Eq. (13) is diagonalized. We introduce the following unitary operators (Schwinger, 1965; Peng and Li, 1996; Cui, 2000)

$$U(\varphi) = \exp \varphi(a_1^+ a_2 - a_2^+ a_1), \tag{14}$$

$$B(\theta) = \exp(-b_1^+ b_2^+ th\theta) \cdot \exp[(b_1^+ b_1 + b_2^+ b_2 + 1) \ln \sec h\theta] \cdot \exp(b_1 b_2 th\theta), \tag{15}$$

$$S_\mu(\xi_\mu) = \exp \frac{1}{2}\xi_\mu \left(e_\mu^2 - e_\mu^{+2}\right), \ \mu = 1, 2. \tag{16}$$

Unitary transformation orderly

$$b_u = U(\varphi) a_u U^+(\varphi), \ e_u = B(\theta) b_u B^+(\theta), \ d_\mu = S_\mu\left(\xi_\mu\right) e_\mu S_\mu^+\left(\xi_\mu\right). \ (\mu = 1, 2). \tag{17}$$

From Eq. (11),(14),(15) and (16), using the following formula

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2!}[A, [A, B]] + \frac{1}{3!}[A, [A, [A, B]]] + \cdots,$$

we can easily prove the following relation

$$b_1 = a_1 \cos\varphi - a_2 \sin\varphi, \quad b_2 = a_2 \cos\varphi + a_1 \sin\varphi; \tag{18a}$$

$$e_1 = \cosh\theta b_1 + \sinh\theta b_2^+, \quad e_2 = \cosh\theta b_2 + \sinh\theta b_1^+; \tag{18b}$$

$$d_1 = \cosh\xi_1 e_1 + \sinh\xi_1 e_1^+, \quad d_2 = \cosh\xi_2 e_2 + \sinh\xi_2 e_2^+. \tag{18c}$$

Substituting Eq. (18) into Eq. (13) and defining zeroth coefficient of$\left(b_1^+ b_2 + b_2^+ b_1\right)$, $\left(e_1^+ e_2^+ + e_1 e_2\right)$, $\left(d_1^2 + d_1^{+2}\right)$, $\left(d_2^2 + d_2^{+2}\right)$, the quantum Hamiltonian of this system is

$$H = A_1'' d_1^+ d_1 + A_2'' d_2^+ d_2 + D'', \tag{19}$$

where

$$A_1'' = A_1' \cosh 2\xi_1 - 2B_2' \sinh 2\xi_1, \ A_2'' = A_2' \cosh 2\xi_2 + 2B_2' \sinh 2\xi_2, \tag{20a}$$

$$D'' = D' + B_2'(\sinh 2\xi_2 - \sinh 2\xi_1) + A_1' \sinh^2\xi_1 + A_2' \sinh^2\xi_2. \tag{20b}$$

$$A_1' = A_1 \cosh^2\theta + A_2 \sinh^2\theta - B_1 \sinh 2\theta,$$

$$A_2' = A_1 \sinh^2\theta + A_2 \cosh^2\theta - B_1 \sinh 2\theta, \tag{21a}$$

$$B_2' = -\frac{\rho}{\sqrt{L_1 L_2 \omega_1 \omega_2}} \frac{\hbar}{4} \sin 2\varphi,$$

$$D' = (A_1 + A_2) \sinh^2\theta - B_1 \sinh 2\theta + \frac{\hbar}{2}(\omega_1 + \omega_2). \tag{21b}$$

$$A_1 = \hbar\omega_1 \cos^2\varphi + \hbar\omega_2 \sin^2\varphi - \frac{\rho}{\sqrt{L_1 L_2 \omega_1 \omega_2}} \frac{\hbar}{2} \sin 2\varphi, \tag{22a}$$

$$A_2 = \hbar\omega_1 \sin^2\varphi + \hbar\omega_2 \cos^2\varphi + \frac{\rho}{\sqrt{L_1 L_2 \omega_1 \omega_2}} \frac{\hbar}{2} \sin 2\varphi, \tag{22b}$$

$$B_1 = \frac{\rho}{\sqrt{L_1 L_2 \omega_1 \omega_2}} \frac{\hbar}{2} \cos 2\varphi, \tag{22c}$$

and $tg 2\varphi = \frac{\rho}{(\omega_2-\omega_1)\sqrt{L_1 L_2 \omega_1 \omega_2}}, \quad \tanh 2\theta = \frac{\omega_2-\omega_1}{(\omega_1+\omega_2)} \sin 2\varphi$

$$\tanh 2\xi_1 = \frac{2B_2'}{A_1'}, \ \tanh 2\xi_2 = -\frac{2B_2'}{A_2'}.$$

Eq. (19) has reduced to diagonalized boson systematic Hamiltonian of non-coupled. Because of unitary transformation of Eq. (17), the energy eigenvalue of this system is fixedness. Therefore, the eigenvalue energy spectrum of mesoscopic damped double resonance inductance coupled circuit is

$$E_{n_1,n_2} = A_1'' n_1 + A_2'' n_2 + D'', \ (n_1, n_2 = 0, 1, 2, \cdots). \tag{23}$$

4. THE QUANTUM FLUCTUATIONS OF THE CHARGE AND CURRENT

Now, we study the quantum fluctuations of the charge and current in excitation state of the squeezed vacuum state. When electrical source is switched off, for the commutation relations defining $[d_1, d_1^+] = [d_2, d_2^+] = 1$, it is assumed to be in excitation state of the squeezed vacuum state, which takes the form (Zhang *et al.*, 1998)

$$|0,0\rangle_{r_1,r_2} = D_1 \operatorname{sec}h^{1/2} r_1 \sum_{n=0}^{\infty} \frac{\left(-e^{j\theta_1} \tanh r_1\right)^n [(2n)!]^{1/2}}{n! 2^n} \sqrt{\frac{(2n)!}{(2n+k_1)!}} \, |2n+k_1\rangle \otimes$$

$$D_2 \operatorname{sec}h^{1/2} r_2 \sum_{n=0}^{\infty} \frac{\left(-e^{j\theta_2} \tanh r_2\right)^n [(2n)!]^{1/2}}{n! 2^n} \sqrt{\frac{(2n)!}{(2n+k_2)!}} \, |2n+k_2\rangle \, . \tag{24}$$

Where, the $k_u, \ r_u, \ \theta_u \ (u = 1, 2)$ stand for quantum number of excitation, squeezed coefficients and squeezed angle, respectively; Canonical coefficient $D_u (u = 1, 2)$ are

$$|D_u|^2 = \left[\sec h r_u \sum_{n=0}^{\infty} \frac{(\tanh r_u)^{2n} [(2n)!]^2}{(n!)^2 \, 2^{2n} \, (2n+k_u)!} \right]^{-1} \quad (u = 1, 2).$$

From Eq. (10) and (18), we obtain

$$Q_1 = \sqrt{\frac{\hbar}{2L_1\omega_1}}\left[\alpha_-\gamma_-\left(d_1 + d_1^+\right) - \beta_-\sigma_-\left(d_2 + d_2^+\right)\right], \tag{25a}$$

$$Q_2 = \sqrt{\frac{\hbar}{2L_2\omega_2}}\left[-\beta_+\gamma_-\left(d_1 + d_1^+\right) + \alpha_+\sigma_-\left(d_2 + d_2^+\right)\right], \tag{25b}$$

$$P_1 = -j\sqrt{\frac{L_1\omega_1\,\hbar}{2}}\left[\alpha_+\gamma_+\left(d_1 - d_1^+\right) + \beta_+\sigma_+\left(d_2 - d_2^+\right)\right], \tag{25c}$$

$$P_2 = -j\sqrt{\frac{L_2\omega_2\,\hbar}{2}}\left[\beta_-\gamma_+\left(d_1 - d_1^+\right) + \alpha_-\sigma_+\left(d_2 - d_2^+\right)\right], \tag{25d}$$

where

$\alpha_\pm = \cos\varphi\cosh\theta \pm \sin\varphi\sinh\theta$, $\beta_\pm = \cos\varphi\sinh\theta \pm \sin\varphi\cosh\theta$, $\gamma_\pm = \cosh\xi_1 \pm \sinh\xi_1$, $\sigma_\pm = \cosh\xi_2 \pm \sinh\xi_2$.

From Eq. (25), we obtain

$$\begin{aligned} {}_{r_1,r_2}\langle 0,0|\, Q_1\, |0,0\rangle_{r_1,r_2} &= 0, \\ {}_{r_1,r_2}\langle 0,0|\, Q_1^2\, |0,0\rangle_{r_1,r_2} &= \frac{\hbar}{2L_1\omega_1}\left(\alpha_-^2\gamma_-^2 x_1 + \beta_-^2\sigma_-^2 x_2\right), \end{aligned} \tag{26a}$$

$$\begin{aligned} {}_{r_1,r_2}\langle 0,0|\, Q_2\, |0,0\rangle_{r_1,r_2} &= 0, \\ {}_{r_1,r_2}\langle 0,0|\, Q_2^2\, |0,0\rangle_{r_1,r_2} &= \frac{\hbar}{2L_2\omega_2}\left(\beta_+^2\gamma_-^2 x_1 + \alpha_+^2\sigma_-^2 x_2\right), \end{aligned} \tag{26b}$$

$$\begin{aligned} {}_{r_1,r_2}\langle 0,0|\, P_1\, |0,0\rangle_{r_1,r_2} &= 0, \\ {}_{r_1,r_2}\langle 0,0|\, P_1^2\, |0,0\rangle_{r_1,r_2} &= \frac{\hbar L_1\omega_1}{2}\left(\alpha_+^2\gamma_+^2 y_1 + \beta_+^2\sigma_+^2 y_2\right), \end{aligned} \tag{26c}$$

$$\begin{aligned} {}_{r_1,r_2}\langle 0,0|\, P_2\, |0,0\rangle_{r_1,r_2} &= 0, \\ {}_{r_1,r_2}\langle 0,0|\, P_2^2\, |0,0\rangle_{r_1,r_2} &= \frac{\hbar L_2\omega_2}{2}\left(\beta_-^2\gamma_+^2 y_1 + \alpha_-^2\sigma_+^2 y_2\right). \end{aligned} \tag{26d}$$

where

$$x_\mu = |D_u|^2\,\mathrm{sech}\, r_\mu \left\{ \sum_{n=0}^{\infty} \frac{[\tanh r_\mu]^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k_u)!}[4n + 2k_u + 1 \right.$$
$$\left. -2(2n+1)\cos\theta_u \tanh r_\mu] \right\},\ \mu = 1, 2;$$

$$y_\mu = |D_u|^2 \operatorname{sech} r_\mu \left\{ \sum_{n=0}^{\infty} \frac{[\tanh r_\mu]^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k_u)!}[4n+2k_u+1 \right.$$
$$\left. + 2(2n+1)\cos\theta_u \tanh r_\mu] \right\}, \mu = 1, 2.$$

Therefore, from Eq. (5) and (26), the quantum fluctuations of the charge and current are

$$\left\langle (\Delta q_1)^2 \right\rangle = \frac{\hbar}{2L_1\omega_1}\left(\alpha_-^2\gamma_-^2 x_1 + \beta_-^2\sigma_-^2 x_2\right)\tau^{-1}\exp(-\lambda t), \tag{27a}$$

$$\left\langle (\Delta q_2)^2 \right\rangle = \frac{\hbar}{2L_2\omega_2}\left(\beta_+^2\gamma_-^2 x_1 + \alpha_+^2\sigma_-^2 x_2\right)\tau^{-1}\exp(-\lambda t), \tag{27b}$$

$$\left\langle (\Delta p_1)^2 \right\rangle = \frac{L_1\hbar}{2\omega_1}\left[\left(\alpha_+^2\gamma_+^2 y_1 + \beta_+^2\sigma_+^2 y_2\right)\omega_1^2 + \frac{\lambda^2}{4}\left(\alpha_-^2\gamma_-^2 x_1 + \beta_-^2\sigma_-^2 x_2\right)\right.$$
$$\left. + 2\lambda\omega_1 z_1\right]\tau^{-1}\exp(-\lambda t), \tag{27c}$$

$$\left\langle (\Delta p_2)^2 \right\rangle = \frac{L_2\hbar}{2\omega_2}\left[\left(\beta_-^2\gamma_+^2 y_1 + \alpha_-^2\sigma_+^2 y_2\right)\omega_2^2 + \frac{\lambda^2}{4}\left(\beta_+^2\gamma_-^2 x_1 + \alpha_+^2\sigma_-^2 x_2\right)\right.$$
$$\left. + 2\lambda\omega_2 z_2\right]\tau^{-1}\exp(-\lambda t). \tag{27d}$$

where

$$z_1 = \left\{ \alpha_+\alpha_-\gamma_+\gamma_- |D_1|^2 \operatorname{sech} r_1 \sum_{n=0}^{\infty} \frac{(\tanh r_1)^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k_1)!} 2(2n+1)\sin\theta_1 \tanh r_1 \right.$$
$$\left. - \beta_+\beta_-\sigma_+\sigma_- |D_2|^2 \operatorname{sech} r_2 \sum_{n=0}^{\infty} \frac{(\tanh r_2)^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k_2)!} 2(2n+1)\sin\theta_2 \tanh r_2 \right\},$$

$$z_2 = \left\{ \alpha_+\alpha_-\sigma_+\sigma_- |D_2|^2 \operatorname{sech} r_2 \sum_{n=0}^{\infty} \frac{(\tanh r_2)^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k_2)!} 2(2n+1)\sin\theta_2 \tanh r_2 \right.$$
$$\left. - \beta_+\beta_-\gamma_+\gamma_- |D_1|^2 \operatorname{sech} r_1 \sum_{n=0}^{\infty} \frac{(\tanh r_1)^{2n}[(2n)!]^2}{(n!)^2 2^{2n}(2n+k_1)!} 2(2n+1)\sin\theta_1 \tanh r_1 \right\}.$$

When $k_u = 0$ $(u = 1, 2)$, the circuit will evolve to squeezed vacuum state from its initial excitation state of squeezed vacuum state. The quantum fluctuations of the charge and current in squeezed vacuum state are

$$\left\langle (\Delta q_1)^2 \right\rangle = \frac{\hbar}{2L_1\omega_1}\left(\alpha_-^2\gamma_-^2 x_1' + \beta_-^2\sigma_-^2 x_2'\right)\tau^{-1}\exp\left(-\lambda t\right), \tag{28a}$$

$$\left\langle (\Delta q_2)^2 \right\rangle = \frac{\hbar}{2L_2\omega_2}\left(\beta_+^2\gamma_-^2 x_1' + \alpha_+^2\sigma_-^2 x_2'\right)\tau^{-1}\exp\left(-\lambda t\right), \tag{28b}$$

$$\left\langle (\Delta p_1)^2 \right\rangle = \frac{L_1\hbar}{2\omega_1}\left[\left(\alpha_+^2\gamma_+^2 y_1' + \beta_+^2\sigma_+^2 y_2'\right)\omega_1^2 + \frac{\lambda^2}{4}\left(\alpha_-^2\gamma_-^2 x_1' + \beta_-^2\sigma_-^2 x_2'\right)\right.$$
$$\left. + 2\lambda\omega_1 z_1'\right]\tau^{-1}\exp\left(-\lambda t\right), \tag{28c}$$

$$\left\langle (\Delta p_2)^2 \right\rangle = \frac{L_2\hbar}{2\omega_2}\left[\left(\beta_-^2\gamma_+^2 y_1' + \alpha_-^2\sigma_+^2 y_2'\right)\omega_2^2 + \frac{\lambda^2}{4}\left(\beta_+^2\gamma_-^2 x_1' + \alpha_+^2\sigma_-^2 x_2'\right)\right.$$
$$\left. + 2\lambda\omega_2 z_2'\right]\tau^{-1}\exp(-\lambda t). \tag{28d}$$

Where

$$x_\mu' = \left|D_u'\right|^2 \mathrm{sech} r_\mu \sum_{n=0}^{\infty} \frac{\left(\tanh r_\mu\right)^{2n}(2n)!}{(n!)^2\, 2^{2n}}\left[\left(4n+1-2\left(2n+1\right)\cos\theta_u \tanh r_\mu\right)\right],$$
$$\mu = 1, 2;$$

$$y_\mu' = \left|D_u'\right|^2 \mathrm{sech} r_\mu \sum_{n=0}^{\infty} \frac{\left(\tanh r_\mu\right)^{2n}(2n)!}{(n!)^2\, 2^{2n}}\left[(4n+1) + 2\left(2n+1\right)\cos\theta_u \tanh r_\mu\right],$$
$$\mu = 1, 2.$$

$$z_1' = \left\{\alpha_+\alpha_-\gamma_+\gamma_- \left|D_1'\right|^2 \mathrm{sech}\, r_1 \sum_{n=0}^{\infty} \frac{(\tanh r_1)^{2n}(2n)!}{(n!)^2\, 2^{2n}} 2\left(2n+1\right)\sin\theta_1 \tanh r_1\right.$$
$$\left. - \beta_+\beta_-\sigma_+\sigma_- \left|D_2'\right|^2 \mathrm{sech}\, r_2 \sum_{n=0}^{\infty} \frac{(\tanh r_2)^{2n}(2n)!}{(n!)^2\, 2^{2n}} 2\left(2n+1\right)\sin\theta_2 \tanh r_2\right\},$$

$$z_2' = \left\{\alpha_+\alpha_-\sigma_+\sigma_- \left|D_2'\right|^2 \mathrm{sech}\, r_2 \sum_{n=0}^{\infty} \frac{(\tanh r_2)^{2n}(2n)!}{(n!)^2\, 2^{2n}} 2\left(2n+1\right)\sin\theta_2 \tanh r_2\right.$$

$$- \beta_+\beta_-\gamma_+\gamma_- \left|D_1'\right|^2 \operatorname{sech} r_1 \sum_{n=0}^{\infty} \frac{(\tanh r_1)^{2n}\,(2n)!}{(n!)^2\,2^{2n}} 2\,(2n+1)\sin\theta_1 \tanh r_1 \Bigg\}.$$

Here $\left|D_u'\right|^2 = \left[\operatorname{sech} r_u \sum_{n=0}^{\infty} \frac{(\tanh r_u)^{2n}(2n)!}{(n!)^2 2^{2n}}\right]^{-1}$ $(u = 1, 2)$
When $r_\mu = 0$ and $k_u = 0$ $(u = 1, 2)$, then $x_\mu' = y_\mu' = 1$ and $z_1' = z_2' = 0$, the circuit will evolve to vacuum state from its squeezed vacuum state. The quantum fluctuations of the charge and current in vacuum state are

$$\left\langle(\Delta q_1)^2\right\rangle = \frac{\hbar}{2L_1\omega_1}\left(\alpha_-^2\gamma_-^2 + \beta_-^2\sigma_-^2\right)\tau^{-1}\exp(-\lambda t), \tag{29a}$$

$$\left\langle(\Delta q_2)^2\right\rangle = \frac{\hbar}{2L_2\omega_2}\left(\beta_+^2\gamma_-^2 + \alpha_+^2\sigma_-^2\right)\tau^{-1}\exp(-\lambda t), \tag{29b}$$

$$\left\langle(\Delta p_1)^2\right\rangle = \frac{L_1\,\hbar}{2\omega_1}\left[\left(\alpha_+^2\gamma_+^2 + \beta_+^2\sigma_+^2\right)\omega_1^2 + \frac{\lambda^2}{4}\left(\alpha_-^2\gamma_-^2 + \beta_-^2\sigma_-^2\right)\right]\tau^{-1}\exp(-\lambda t), \tag{29c}$$

$$\left\langle(\Delta p_2)^2\right\rangle = \frac{L_2\,\hbar}{2\omega_2}\left[\left(\beta_-^2\gamma_+^2 + \alpha_-^2\sigma_+^2\right)\omega_2^2 + \frac{\lambda^2}{4}\left(\beta_+^2\gamma_-^2 + \alpha_+^2\sigma_-^2\right)\right]\tau^{-1}\exp(-\lambda t). \tag{29d}$$

When $C \to \infty$, $r_\mu = 0$ and $k_u = 0$ $(u = 1, 2)$, then $\alpha_\pm = 1$, $\gamma_\pm = 1$, $\sigma_\pm = 1$, $\beta_\pm = 0$, we can obtain

$$\left\langle(\Delta q_1)^2\right\rangle = \frac{\hbar}{2L_1\omega_1'}\tau^{-1}\exp(-\lambda t), \quad \left\langle(\Delta q_2)^2\right\rangle = \frac{\hbar}{2L_2\omega_2'}\tau^{-1}\exp(-\lambda t), \tag{30a}$$

$$\left\langle(\Delta p_1)^2\right\rangle = \frac{L_1\,\hbar}{2\omega_1'}\left(\omega_1'^2 + \frac{\lambda^2}{4}\right)\tau^{-1}\exp(-\lambda t),$$

$$\left\langle(\Delta p_2)^2\right\rangle = \frac{L_2\,\hbar}{2\omega_2'}\left(\omega_2'^2 + \frac{\lambda^2}{4}\right)\tau^{-1}\exp(-\lambda t). \tag{30b}$$

Where $\omega_\mu' = \sqrt{\frac{1}{L_\mu C_\mu} - \frac{\lambda^2}{4}}$, $\mu = 1, 2$. Eq. (30) reveals that there are quantum fluctuations of the charge and current in mutual independency damped harmonic oscillator. This squeezing originates from the coupling effect and damping. So the uncertainty relation is

$$\langle(\Delta q_\mu)^2\rangle\langle(\Delta p_\mu)^2\rangle = \frac{\hbar^2}{4\omega_\mu'^2}\tau^{-2}\omega_{0\mu}^2\exp(-2\lambda t). \tag{31}$$

Where$\omega_{0\mu} = \sqrt{1/L_\mu C_\mu}$, $\mu = 1, 2$. This is direct result of the commutation relations Eq. (4). Set $R_1 \to 0$, $R_2 \to 0$, we have the quantum fluctuations of two independent non-damped harmonic oscillator

$$\left\langle (\Delta q_1)^2 \right\rangle = \frac{\hbar}{2L_1\omega_{01}}\tau^{-1}, \quad \left\langle (\Delta q_2)^2 \right\rangle = \frac{\hbar}{2L_2\omega_{02}}\tau^{-1}, \tag{32a}$$

$$\left\langle (\Delta p_1)^2 \right\rangle = \frac{L_1\,\hbar\omega_{01}}{2}\tau^{-1}, \quad \left\langle (\Delta p_2)^2 \right\rangle = \frac{L_2\,\hbar\omega_{02}}{2}\tau^{-1}. \tag{32b}$$

From the above equations, there are quantum fluctuations of the charges and current in mesoscopic damped double resonance capacitance coupled circuit. The squeezed vacuum state and the vacuum state can be regarded as special cases of the excitation state of the squeezed vacuum state. This quantum fluctuations are related not only to circuit device parameter and coupling magnitude of mutual capacitance, but also quantum number of excitation, squeezed coefficients and squeezed angle, and because of damped resistance R_1, R_2 influence, the quantum fluctuation of the charges and current decay along with time.

5. CONCLUSIONS

In this letter, from the classical equations of motion, mesoscopic damped double resonance mutual capacitance coupling circuit was quantized by the method of damped harmonic oscillator quantization, Hamiltonian was diagonalized by the method of thrice unitary transformation. The energy eigenspectra of this circuit were given. We have studied the quantum fluctuations of the charges and current of each loop. The result indicate that (1) when mesoscopic damped double resonance mutual capacitance coupled circui is quantized, It is equivalent with quantized double coupled harmonic oscillator. (2) There are quantum fluctuations of the charge and current of each loops. The squeezed vacuum state and the vacuum state can be regarded as special cases of the excitation state of the squeezed vacuum state. This quantum fluctuations are related to not only circuit device parameter and coupling magnitude of mutual capacitance, but also quantum number of excitation, squeezed coefficients, squeezed angle and damped resistance R_1, R_2. Because of ullage resistance R_1, R_2, the quantum fluctuation of the charges and current decay along with time. (3) When coupling capacitance $C \to \infty$, it will evolve to two mutual independent damped harmonic oscillator. When $C \to \infty$ and $R_1 \to 0$, $R_2 \to 0$, it will evolve to two non-coupling and non-damped harmonic oscillator, the quantum fluctuations is related to inherent resonance frequencies. The circuit quantum noise can be controled by adjusting device parameters. For mass circuit, it is damped. Therefore, studying quantum effects of the mesoscopic damped double resonance mutual capacitance coupling circuit is more significant to not only design and apply practical mesoscopic circuit but also academic guidance.

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